

Virtual flowers generated by Generalized Rosy Curves¹

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Abstract

The properties and applications of rosy curve are introduced first in the paper. Afterwards, a class of generalized rosy curves is proposed. Their algebraic and geometric properties are discussed and their applications are prospected. Then, they are used to represent virtual flowers of different types

1. Introduction

Curves are the basic class of elements in computer graphics. With the development of computer graphics in recent decades, many kinds of curves have been proposed and applied widely in fields such as scientific computation, visualization of statistics data, shape design of auto and aircraft, etc.

The most mainly used curves at present are piecewise or rational polynomial expression with parameters. Besides, there are many other forms to define curves and surfaces: Ferguson (1963) used parametric vector functions to describe curves and defined a piece of surface with four angular points and two tangent vectors; Coons(1964) used four boundaries of closed curve to define surface; Schoenberg(1964) proposed parametric spline forms of curves and surfaces; Bezier(1971) used control polygon to define curves and surfaces; De Boor(1972) put forward the standard computation method of B-spline. Afterwards other researchers proposed rational B-spline and the method of none-uniform rational B-spline (NURBS) which have been become the most popular technique to describe free curves and surfaces nowadays. We can use NURBS to represent uniformly the elementary analytical curves, surfaces, rational or non-rational Bezier curves and non-rational B-spline curves.

Rosy curves discussed in this paper belong to a class of non-polynomial parametric curves with polar coordinate representation and they have not been

applied yet in compute graphics. We will investigate the curves' shape, properties and their algorithm of generation and transformation in the following sections. The form of generalized rosy curve and equation of 3D rosy curve are also proposed in the end.

2. Standard rosy curve's expression and its geometric structure

All printed material, including text, illustrations, and charts, must be kept within a print area of 6-1/2 inches (16.51 cm) wide by 8-7/8 inches (22.51 cm) high. Do not write or print anything outside the print area. All *text* must be in a two-column format. Columns are to be 3-1/16 inches (7.85 cm) wide, with a 3/8 inch (0.81 cm) space between them. Text must be fully justified.

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In general the polar representation of rosy curve is given as:

$$\rho = A + B \cos(M\theta) \quad (1)$$

Where A, B are real constants and M is a positive integer.

From eq(1) we can obtain immediately that rosy curve has the periodic property and its envelope line is a circle. The curve's shape likes a rose just as its name, especially 5-leaves rosy curve. The parameters in eq(1) are A, B and M , and they have respective effect to control the shape of rosy curves:

Parameter A: determine the heart location of flower and the width of leaves;

Parameter B: determine the length of leaves;

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Parameter M: determine the number of leaves

Some images of simple rosy curves are given in the following figure 1:

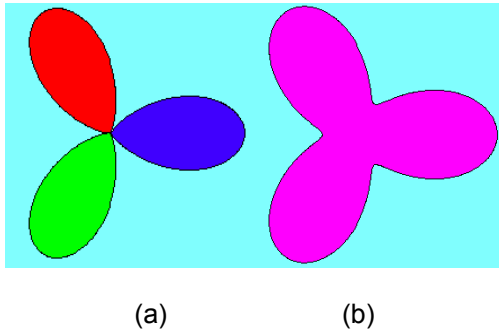


Figure 1 Simple rosy curves

The two rosy curves in figure 1 correspond to $M = 3$ such that they have three leaves. In figure 1(a) $A = B$ and in figure 1(b) $A = 1.5B$. We can see the bigger A is, the fatter leaves become.

When we set $M = 5$, the image is:

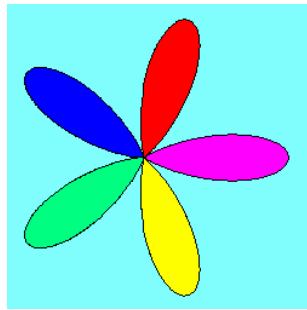


Figure 2 $M = 5$, $A = B$

We can see the number of leaves turns from three to five.

3. The basic properties of rosy curves

Now we will discuss the properties of rosy curves in detail. From equation (1) the range of ρ is $[A-B, A+B]$. Because parameters (A, B) only determine the measurements of curves, whether they are real numbers or integers the effects to basic shape of rosy curves are similar.

3.1. The effects of parameter A, B to the shape of curves:

3.1.1. $A = B$

Each leaf grows from the heart of flower and connects to a circle whose diameter is $A + B$. As shown in figure 3(a).

3.1.2. $A > B$

Each leaf grows from a circle whose diameter is $A - B$ and connects to another circle whose diameter is $A + B$. The two circles have the same center. As shown in figure 3(b).

3.1.3. $A < B$

In this case ρ might be a negative number and its minimum is $A - B$. The leaves are symmetrical about the heart and they include a group of small ones whose length is $B - A$. The small leaves connect to a circle whose diameter is $B - A$. And the big leaves connect to a circle whose diameter is $A + B$. As shown in figure 3(c).

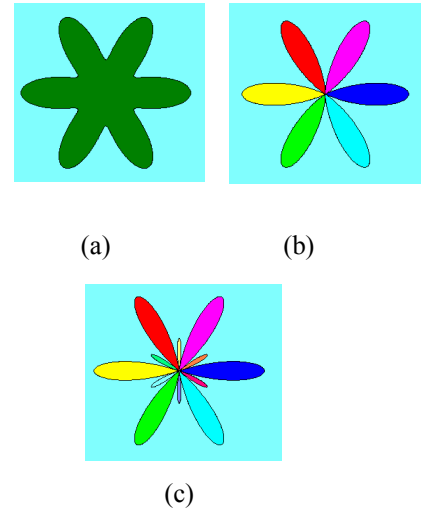


Figure 3 The effects of parameter A, B to the shape of rosy curves

3.2 The effects of parameter M to the shape of curves:

From $\cos(-\theta) = \cos \theta$ we can get whether M is a positive or negative number the curve is the same one. Therefore we discuss the case $M > 0$ only:

3.2.1. $M = 0$:

Equation (1) becomes:
 $\rho = A + B \cos 0 = A + B$, the curve reduces to a circle.

3.2.2. $M = 1, 2, \dots$

M is the number of leaves. The rosy curve is closed when θ turns from 0 to 2π and the leaves don't intersect from each other. Figure 4 is the image of rosy curve when $M = 15$.

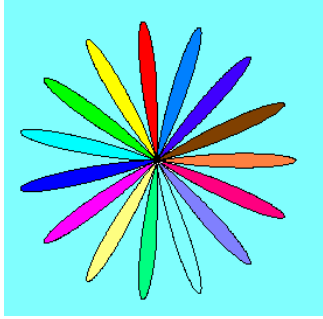


Figure 4 $M = 15$

Especially when $M = 1$ the curve has only one leaf and it reduces to heart curve. As shown in figure 5.

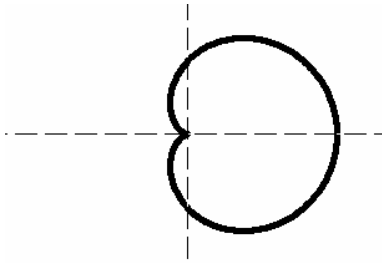


Figure 5 Heart curve

4. Generalization of standard rosy curves

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In standard rosy curves M is an integer, now we investigate the case that M is a non-integer number. At first assume M is a rational number, that is, $M = P/Q$, where P and Q are positive integers. P determines the number of leaves and Q the width of leaf or the number of intersecting points. In this case rosy curves are also closed when θ turns from 0 to $Q \cdot 2\pi$ which is bigger than the case M is an integer.

When M is greater than 1 and less than 1 the shapes of rosy curves are different.

4.1 when $P > Q$ (e.g. $P = 5, Q = 3$)

the curve is shown as figure 6:

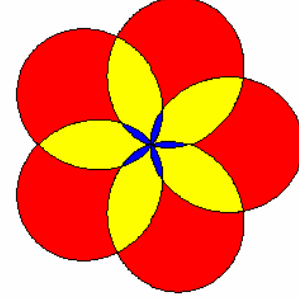


figure 6 M is a rational number ($P > Q$)

In the image there exist five leaves because $P = 5$ and the number of intersecting points is three because $Q = 3$. A big leaf is divided into three small leaves. In the whole image pushed-down three simple rosy curves with three stuffed colors.

4.2 when $P < Q$ (e.g. $P = 3, Q = 5$)

the curve is shown as figure 7:

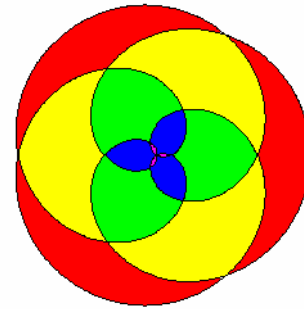


figure 7 M is a rational number ($P < Q$)

In the image there exist three leaves because $P = 3$ and the number of intersecting points is five because $Q = 5$. A big leaf is divided into five small leaves. In the whole image pushed-down five simple rosy curves with five stuffed colors.

From the above two cases, when $P > Q$ the curve likes rose flower much more and when $P < Q$ the curve tends to a circle. Figure 8 is the image of rosy curve when $M = 1/5$:

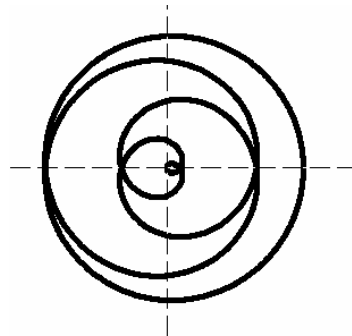


Figure 8 $M = 1/5$

In figure 9 are some other more complicated rosy curves' images:

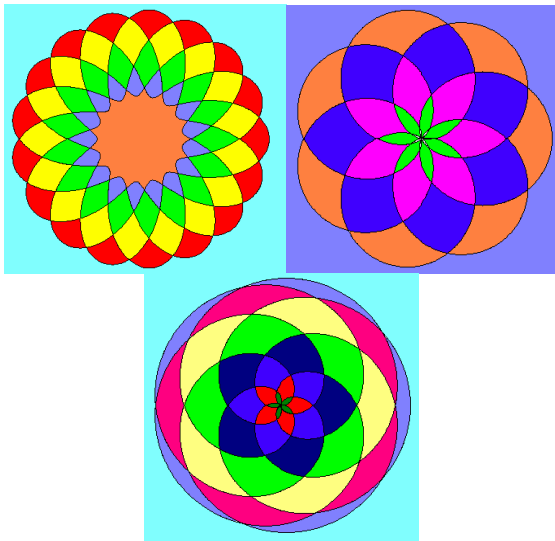


Figure 9 Complicated rosy curves

5. Conclusion

Because of the concise equation and the changeful shapes rosy curve can be applied in many fields. We can use rosy curve to compress images for its simplicity of representation, to form the flower pattern on decorative clothing for its beauty of shape, to generate anti-fake label for its diversity and difficulty to imitate. Besides we can generalize rosy curves to higher-order forms

6. References

List and number all bibliographical references in 9-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example [1]. Where appropriate, include the name(s) of editors of referenced books.

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