# Visualization of Dynamic Network Evolution With Quantification of Node Attributes

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***Abstract*—The effective visual exploration of dynamic networks has been one of the toughest challenges and an unsolved problem; however, it is very important to understand network evolution. Although many developments have been achieved in modeling evolutionary networks, the closely related task of visualizing continues to remain a major concern. Therefore, in this study, quantitative analysis is used to assign node attributes in the network topology, and then, the evolutionary process of networks is analyzed. By fixing the position of nodes, the possibility of a stationary shape of the network is suggested, and a more intuitive and comprehensive explanation of the enumeration of the types for the evolutionary process is provided. Further, a large amount of information is presented in this study in an extremely economical and accessible way by incorporating a circular layout and evolution laws, which offers a new approach for the estimation and evaluation of network evolution. Finally, this three-pronged approach—network analysis, quantitative method, and topological modeling—is expected to provide a revelatory insight into the principle of network evolution.**

***Index Terms*—Art, Communication/Networking and Information Technology, Data and Knowledge visualization, Evolution, Image Processing and Computer Vision, Simulation, Modeling, and Visualization.**

## I. Introduction

The wealth of data presents new opportunities for theorists, requires the refinement of statistical and modeling techniques, and provides us with insight to question the assumptions of the relationship between network topology and evolution. The effective visual exploration of dynamic networks has been one of the toughest challenges and continues to remain an unsolved problem; however, it is highly important to understand network evolution. This is why many real-world networks, including social and information networks, are dynamic structures that evolve over time because of the creation of new nodes and edges and the removal of old nodes and edges [1]. As dynamic networks have both structural and temporal properties, it is difficult to track changes over time [2], [3]. Although many empirical research studies have focused on statistical mechanisms of evolution such as scale-free networks and small-world networks [4], [5], these approaches require a fair amount of knowledge to interpret the mathematical laws of formulas.

To overcome this limitation, considerable interest has been diverted to modeling less abstract and more intuitive networks. The purpose of modeling a network is to augment the theoretical intuition provided by summary statistics and standard static visualization. Currently, two dominant approaches exist for assessing evolving networks in terms of dynamical and topological properties:

1) An information diffusion algorithm, which is considered as the underlying network dynamics. Recent studies have shown that information diffusion and network evolution are coupled, and that network changes are often triggered by information diffusion [6], [7], epidemic spreading [8], or evolutionary games on interdependent networks [9]. Further, they show experimental evidence that nodes constantly create new links when exposed to new information sources and, in turn, these links alternate the manner in which the information spreads [10], [11], [12]. In addition, different types of information are being studied because people tend to have different attitudes toward information in the real world [13].
2) Graph clustering, which infers the full network topology using different approaches, has received considerable attention in view of evolving networks [14]. Clustering is performed to partition data into different groups such that the nodes have high similarity to each other and reflect the intrinsic structure of the data [15]. Unlike the previous method, which classifies nodes into clusters directly, network embedding and community detection have gained popularity. Community detection obtains the most noticeable cluster or one specific cluster with labels and node attributes. Network embedding, on the other hand, aims to represent nodes by vectors that encode as many diverse properties as possible. Multifacet network embedding (MNE) [16], structural deep network embedding (SDNE) [17], and LINE [18] have been suggested to effectively exploit the most noticeable structure of the network.

These structural viewpoints aim to understand networks not just as topological objects, but also as the framework upon which a distributed dynamic system is built [19]. Nevertheless, the visualizing of dynamic networks is still approached in terms of modeling that is close to the concept of “reading” rather than “seeing” data. The purpose of modeling is to

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explore the data, understand the relationships relevant to the problem at hand, and capture those relationships in a computational model that will match a specific requirement. Modeling is a method to bring flexibility to logical and physical data by redefining and combining some data elements, entities, and relationships within the model into more generic terms [20]. Phrased differently, modeling is to describe the process as consisting of the "context of discovery" and the "context of justification." However, because of its versatility, modeling has been criticized for being too far out of the "intuitive" realm because it is not directly related to the physical perceivable world [21].

To augment theoretical intuition, visualization is emerging as an effective complement to better understand modeling. Visualization enables us to analyze large amounts of data that can hardly be overseen in numerical form and to view the overall structure easily. Translating numerical information visually provides a researcher with a more complete multivariate view, which can enable them to observe how each node is embedded in the global structure and facilitate communication of structural findings to scientific and non-scientific audiences [22]. The ability to see data clearly creates a capacity for building intuition surpassed by summary statistics [23]. In addition, visualization is especially useful for dynamic networks that are represented by multiple facets such as multivariate attributes, or spatial and temporal frames of reference [24]. Selecting various algorithms to map node attributes allows the properties of the network to be studied with respect to the ordering principles of a given layout. When many links prevent people from recognizing salient structural patterns [25], reducing methods such as the minimum spanning tree (MST) and pathfinder network scaling (PFNET) algorithms [26], [27] are also used for effective visualization. These strategies can help identify some sort of backbone for the overall topology and analyze temporal networks, which describe graph changes diachronically. However, the most important aspect is that network topology can directly lead to the capture of evolutionary processes when network modeling and visualization are well integrated. Thus, we propose a novel method that incorporates network modeling and visualization for a more intuitive and comprehensive understanding of network evolution. Through the process of deriving modeling from visualization, we contribute to a more general understanding of the evolutionary network process and dynamics of the network.

We aim to address several issues concerning visualizing the evolution of a network. These issues are brought forward by the following questions: 1) What makes an intuitive visualization of an evolving network? 2) How does a network evolve? 3) Can we find laws and derive models that explain its evolution? 4) What are the implications of our findings on visualizing the evolution of a network in general? To answer these questions, the remainder of this article is organized as follows. A practical method for the quantitative definition of node attributes is proposed, and this method is used to outline the association between quantitative analysis and structural dynamic network patterns. In addition, the evolution laws by Michael Leyton are introduced; these laws are redefined to the analysis of evolving networks in Section 2. Then, the feasibility and reliability of the visualization is verified through the analysis of Dutch artists and French Impressionists networks as a case study in Section 3. The analysis allows us to obtain a basis for the general modeling of network evolution in Section 4. Finally, conclusions and discussions for future work are presented in Section 5.

## II. Method

Networks are characterized as complex systems that comprise many nodes coupled by specific, potentially changing interaction topologies [4]. Therefore, many researchers argue that there is no such thing as "the" network [12]. The shape of a network has been recognized as heterogeneous because it changes according to the attributes and ties between the nodes and, therefore, it has always seemed impossible for the network topology to be determined rigorously and systematically. Although the massive and comparative analysis of networks such as through parallel coordinate plots [28], arc-diagrams [29], or multilayer visualization [30] has already been explored under different application domains, a better approach is to require a static overview of the entire time span of the network. That is why the difficulty in focusing on many properties simultaneously and the difficulty in tracking changes over time spans is an ongoing problem [3]. To solve the problem, it is important to recognize how to position and connect the nodes such that their own attributes are highlighted for the exact visualization of network topology. Therefore, this study attempts to prove that a network can have its own shape, thereby enabling better exploration of network topology.

By fixing nodes and relations in artificial positions, we create a topological network space that can reveal more information about both the evolution of the network and the essence of the nodes that cause the evolution. Basically, visualization is a process that (a) is based on qualitative or quantitative data and (b) results in an image that is representative of the raw data, which is (c) readable by viewers and supports exploration, examination, and communication of the data [31]. Considering this fact, we attempt to offer examples of how data visualization will be used in evaluations to help aid understanding, collect data and information, conduct analysis, and communicate with a variety of scholars. For this purpose, we employ the following process: 1) identify informative nodes, 2) quantitatively define node attributes, 3) quantitatively position the nodes, and 4) apply the evolution laws for defining network topology with optimization of quantitative criteria.

### A. Identifying Informative Nodes

Art movements are large information networks where artists share and communicate information via their signature styles and personal relationships. An artistic network is one of the prime loci of artistic innovation and development, and it is possible to obtain numerical values for identifying node


Fig. 1. Example of conventional visualization in Dutch artists' network.

attributes and cooperative links among artists through quantitative analysis. Each artist's style can be used as the main indicator for investigating the process of information diffusion in the network. The diversity and universality of art—two of the most significant aspects of cultural creativity—can be utilized to explain an evolving dynamic structure that is constructive and a means to analyzing a network through the statistical valuation of artistic style. Thus, artistic networks—specifically that of the Dutch artists and French Impressionists networks—were focused on in this study.

In the case of the Dutch network, nodes were generated by referring to "the register of the Guild of Saint Luke," which several Dutch artists deeply identify with [32]. Focusing on the master–pupil relationships, 144 artists who were active between 1600 and 1700 were collected, and each artist was categorized according to the period of most intense activity. Fig. 1 shows the network visualized using *Gephi*. The text size of the nodes represents their importance and is estimated by degree centrality, which is the standard indicator of the level of centrality or activeness in a network. Next, a total of 21 Dutch artists were selected according to the highest degree centrality as the most influential members of that network, and a total of 903 paintings were collected from these artists.

In the case of the French Impressionists network, 117 artists who were active between 1800 and 1900 were collected based on various historical documents, such as related papers, artists notes, and online art databases. Next, a total of 21 artists were selected to represent the art movement, and a total of 2,447 painting were obtained from these artists. Because all nodes are not significant, the prominent nodes were determined to emphasize either typical or anomalous patterns of the network. This strategy is valid to ensure the reduction of visual clutter, and it enable more efficient use of network space.

### B. Quantitative Definition of Node Attributes

The location of artists and the topological proximity between them were specified using the characteristic style of an artist's paintings expressed in the quantitative form. Because brushstroke is a representative attribute for revealing an artist's habitual painting style, brushstroke analysis was employed to extract the signature styles of these artists. Based on multiscale oriented Gabor wavelet filters, four scales (2, 4, 8, 16) and six orientations (0°, 30°, 60°, 90°, 120°, 150°) were employed to calculate Gabor energy (GE), which is defined as the sum of the squared values obtained by convolving $G_{odd}$ and $G_{even}$; these are given as

$$E^2(x,\ y) = \ {G_0}^2(x,\ y) + \ {G_e}^2(x,\ y) \tag{1}$$

$$G_o(x,y) = \ exp\left[\left(\frac{x^2}{\sigma^2 x} + \frac{y^2}{\sigma^2 y}\right)\right] \times Asin(\omega_s x) \tag{2}$$

$$G_e(x,y) = \ exp\left[\left(\frac{x^2}{\sigma^2 x} + \frac{y^2}{\sigma^2 y}\right)\right] \times Acos(\omega_s x) \tag{3}$$

This value is calculated by averaging across all spatial frequency scales and different orientations, and it is used to derive the representative value of each artist's style [33]. For example, for Paul Cezanne (1839–1906), the number of paintings used in the brushstroke analysis was 230, and therefore, the total number of numerical values extracted from these paintings was 230. However, not all these values represent his style property because most artists sometimes use a unique style of painting, unlike their typical style. Therefore, standard error (SE), which measures the precision of estimates of population mean $\mu$, was applied in this study. The SE of a mean provides a statement of

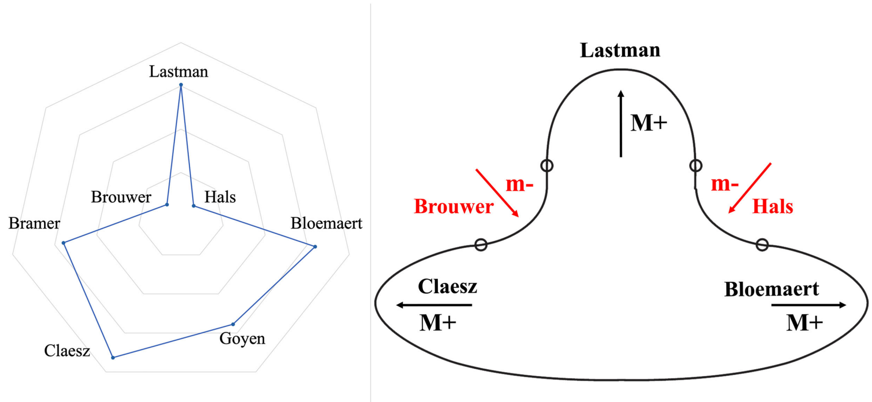


Fig. 2. Circular representation of nodes in a network topology.

probability about the difference between the mean of the population and the mean of the sample, and it allows us to calculate a confidence interval (CI) that defines a range of values that certainly contain the population mean. The CI indicates the level of confidence about the true population mean falling within a given range. Considering a normal distribution, most values lie between $\mu$-2$\sigma$ and $\mu$+2$\sigma$, and the lower bound (4) and upper bound (5) for the mean was calculated as

$$L_{bound} = \left(m - 2\frac{\sigma}{\sqrt{n}}\right), \tag{4}$$

$$U_{bound} = \left(m + 2\frac{\sigma}{\sqrt{n}}\right), \tag{5}$$

where *m*, $\sigma$, and *n* denote the sample mean, population standard deviation, and sample size, respectively. Thus, all values were calculated on the average value that did not exceed $\pm$2SE, and therefore, the result values were used to obtain the mean of the normally distributed values of each artist. The normalized values were converged to one value by taking average values, which were then used to assign the node position of each artist.

### C. Quantitative Positioning of Nodes

To assign the position of nodes in the network topological space, a circular layout was used. The circular view supports the visualization of the changing characteristics over time thereby allowing us to observe changes in the data [34]. It enables us to explore both the temporal and structural features of the network simultaneously (“spatiotemporal mapping”) [3], [35]. Furthermore, it plays an important role in reducing visual clutter. As the volume of data increases, a large number of line crossings and overlaps produce unreadable images. The most effective way to solve the problem is concentric-circle visualization, wherein the axes are organized as concentric circles rather than as line crossings [36], [37], [38]. Because circular layouts have a uniform distribution of node positions and edge crossing angles in circles, the advantage improves the readability of the visualization even though the added data are considerably dense.

This study thus utilized circular layout to track nodes continuously changing over time. The temporal feature is encoded by the radius of the circle. A node appearing at the beginning is drawn on the right side of the circle, whereas a node occurring at the end is rendered at the left side of the circle. In other words, the order in which the nodes appear is presented clockwise. Furthermore, the artists were divided using five-steps, which were based on the period of intense activity, and then the location of each artist was fixed according to their representative values from the quantitative analyses. In each step, about four or five artists were included and the final shape of the network was created by a soft connection between the node’s position, as shown in Fig. 2.

### D. Applying the Evolution Laws

This study introduced the evolution laws by Michael Leyton’s theory for the topological differences of networks. A central part of the theory involves the establishment of the rules by which it is possible to extract the memory from the shape [39]. Four types of curvature extrema intended for describing the evolution laws are recovered from a shape, as listed in Table I.

The evolution laws express any history of shape evolution in terms of the progressive changes of four types of curvature extrema. The “M+” extremum corresponds to a protrusion because it indicates a force going outside from the inside pushing out, whereas the “m-” extremum is an indentation because it shows a force pushing into the inner space from the outside. The “M-” is a resistance because it shows a force coming from the inside and resisting the inward force, whereas the “m+” indicates the force as squashing the inner space [39]. In short, “M+” and “m-” are considered more direct influences on network shape with regard to the penetrating effects of protrusion and indentation, whereas “M-” and “m+” are considered indirect influences because the forces of squashing and resistance cannot immediately change the shape. The former and latter are called bifurcation and continuation, respectively. Continuation relates to the breaking of smoothness called “cusp-formation,” which implies the decline and eventual extinction of a shape; appropriate bifurcation is necessary to maintain the shape.

TABLE I
REDEFINED EVOLUTION LAWS FOR ADJUSTING NETWORK EVOLUTION

| | Extremum | Redefined meaning of extremum | Evolution laws | Redefined evolution laws |
|---|---|---|---|---|
| Continuation | M+ (Protrusion) | Development of style | Cusp-formation | Toward Convergence |
| | M- (Resistance) | Maintenance of style | Change to M+ | |
| | m+ (Squashing) | Introduction of new style | Change to m- | |
| | m- (Indentation) | Inflow of new style | Cusp-formation | |
| Bifurcation | M+ (Protrusion) | Development of style | Two copies of M+ | Toward Divergence |
| | M- (Resistance) | Maintenance of style | No change | |
| | m+ (Squashing) | Introduction of new style | Not change | |
| | m- (Indentation) | Inflow of new style | Two copies of m- | |

According to the characteristics, we redefined the evolution laws to fit the evolution of the artists network as shown in the third column of Table I. First, the "M+" extremum is regarded as a force for the development of artistic style. This outward force corresponds to the environment when certain artistic styles have considerable influence in the world. Second, the "m-" extremum represents a force for the inflow of a new style in terms of creating a new space and causing a bifurcation. This inward force corresponds to the birth of a new culture that started with a unique artist with special way of painting. Third, the "M-" extremum is translated into a force for the maintenance of style because it is an outward force against forces like the "m-" extremum. Although the force is not recognized as being crucial to change the evolution process, it serves the important function of preserving artistic style. Given that the fixed and standardized rules determine what constitutes an art movement, the force is necessary until artists can participate in the group. Finally, the "m+" extremum is translated into a force for the introduction of style because it is an inward force in preparation for the successful inflow of a new style. Based on these laws, we analyzed Dutch artists and French Impressionists networks and demonstrated how each node evolves the entire network through the topological representation of the network structure.

## III. RESULTS

This study examined Dutch artists and French Impressionists networks by applying the process of network evolution. As shown in Figs. 3 and 4, the five-step evolution process begins from a circle to gain an intuitive sense of what part of the shape changes or remains unchanged after a transformation. In geometry, symmetry can be thought of as an immunity to change ("indistinguishability"), whereas asymmetry is understood as having originated from a past symmetry ("distinguishability"). A circle rotated about its center will have the same shape and size as the original circle because all points before and after the transformation are indistinguishable. Therefore, a circle is a highly symmetric shape and has rotational and reflectional symmetry around the center for every angle. Consequently, a circle can be the past of any smooth closed shape and reveal structural information about a network, thereby enabling rapid comparison of networks [30]. Thus, the circle is postulated as the starting point, and the evolutionary history recovered from each step of the circular layout is tracked. In addition, how each artist acted as a means of extremum for the evolution of the entire network is compared in accordance with the records of art history. By doing so, this study proves the feasibility and reliability of the method.

### A. Network Evolution of Dutch Artists

Fig. 3 shows the evolutionary process of "bifurcation–continuation–bifurcation–continuation." Fig. 3(b) shows the process state where the two copies of "m-" are on each side, together with the central "M+" extremum between them. This evolution includes a bifurcation "m-" and a breakthrough where the initial squashing process is pushed to either side by a newly inserted protrusion. In this figure, Pieter Lastman (1583–1633), Leonaert Bramer (1596–1674), and Abraham Bloemaert (1566–1651) are the "M+" extrema. All these artists occupied a place of their own among the artists active in Delft, profoundly affecting upcoming artists such as Rembrandt van Rijn (1606–1669). In view of this fact, the transition from Fig. 3(b) to (c) shows the most influential artists who contributed to the development of Dutch art and the artistic genealogy at that time.

Fig. 3(c) also provides empirical evidence of Rembrandt's importance in Dutch art. Rembrandt had many prominent pupils, and his studio produced many fine artists of the period. Gerrit Dou (1613–1675), Carel Fabritius (1622–1654), Frans van Mieris (1635–1681), and Nicolaes Maes (1634–1694) were notable pupils of Rembrandt. Rembrandt's studio offered a far less rigid apprenticeship than those regulated by art guilds, and pupils were not forced to adopt Rembrandt's manner of painting. For example, Dou and Maes developed

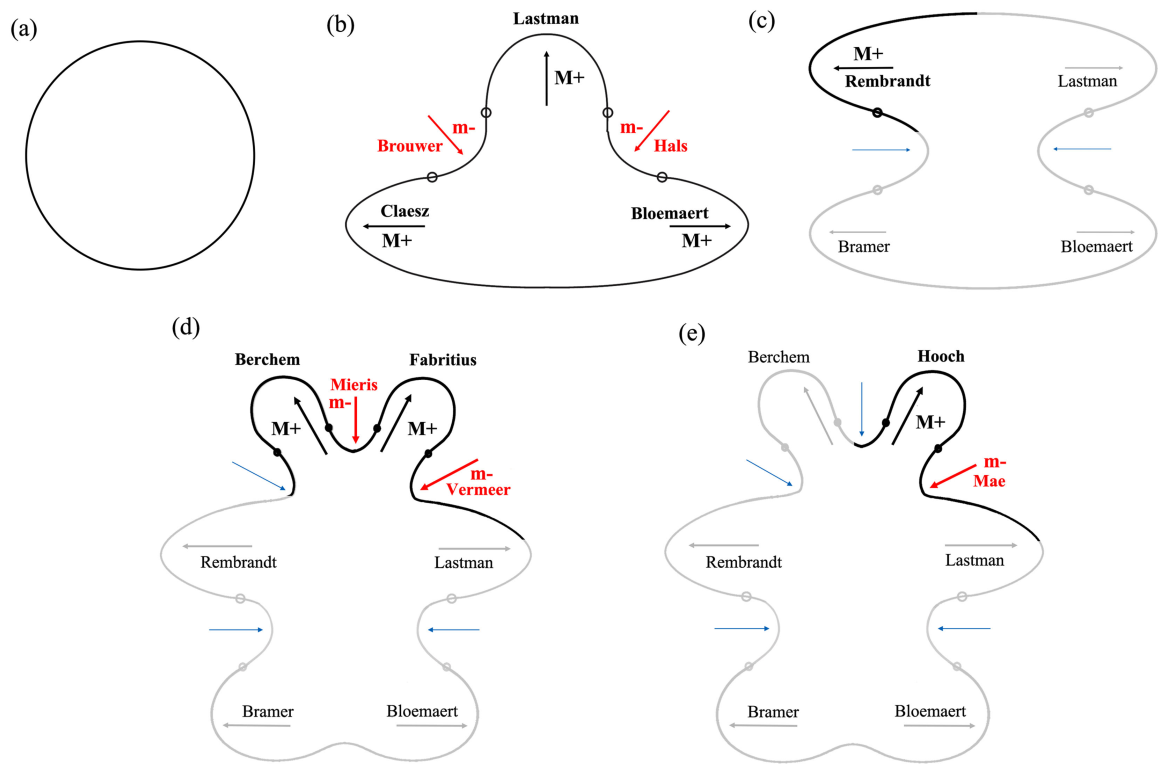


Fig. 3. Five-step network based on brushstroke analysis of Dutch artists. The red arrow indicates inward movement ("m-," indentation), whereas the black arrows signify outward movement ("M+," protrusion); gray arrows show what has occurred in the previous step. Blue arrows affect the shape of the network; however, they are not a key factor in providing a driving force for the evolution of the network.

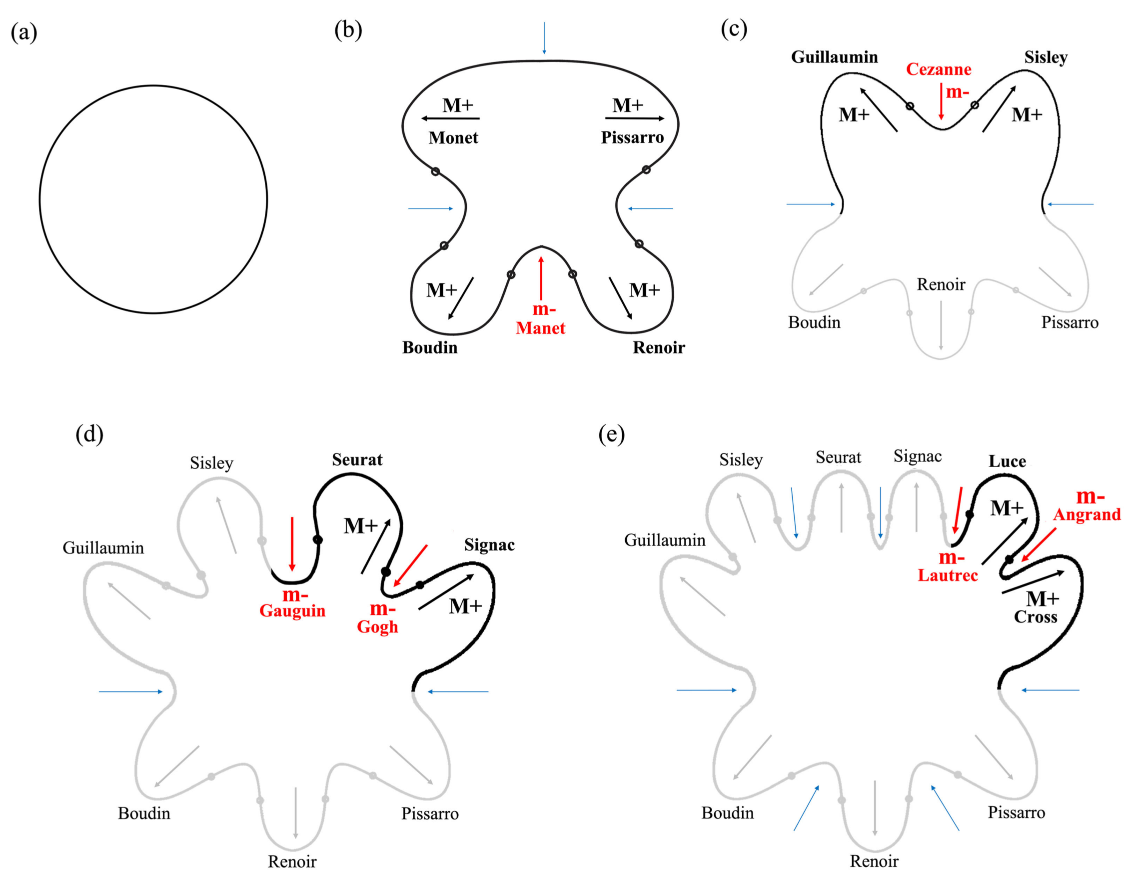


Fig. 4. Five-step network based on brushstroke analysis of French Impressionists.

distinctive styles of their own at a comparatively early point in their careers [41]. Consequently, this apprenticeship contributed to maintain the diversity of styles in art, and it served as the foundation for the Dutch Golden Age.

Fig. 3(d) is a good illustration of the development of Dutch art resulting from the initiative of the pupils of Rembrandt and the inflow of new style by the next generation of artists. In the figure, Nicolaes Berchem (1620–1683) and Fabritius are the “M+” extrema, whereas Johannes Vermeer (1632–1675) and van Mieris are the “m-” extrema. The initially mentioned artists attempted not only to absorb Rembrandt’s style but also develop in their own style. The latter artists were not only the leading force of the Dutch Golden Age, but they also formed a new mainstream represented by the genre painting of Gerard Ter Borch (1617–1681). Ter Borch, who strongly influenced Mieris, Pieter de Hooch (1629–1684), and Vermeer, would have walked in step with those artists as a mediator and mentor [42]. Therefore, Fig. 3(d) shows a circumstance of inflows of new styles of art at that time.

Fig. 3(e) especially shows the crucial wave of de Hooch at that time. The work of de Hooch was imitated in Delft and elsewhere, and it became known as the School of de Hooch, even though he never had any pupils [43]. Meanwhile, Maes dedicated himself almost exclusively to portrait painting at the end of his travel to Antwerp. His stylistic evolution, which was alien to the other mainstream, granted him success as a society portraitist dating back to the 1670s and 1680s. Consequently, Fig. 3 shows two major flows in the growth of Dutch art from 1600 to 1700.

### B. *Network Evolution of French Impressionists*

French Impressionism shows the evolutionary process of bifurcation at each step. In Fig. 4(b), the arrow named Edouard Manet (1832–1883), is an “m-” extremum, which shows a force going into the network from the outside, whereas the other artists show outward movement as a reaction to the penetrative force. Manet had an enormous influence upon his contemporaries; he rejected the label “impressionist,” preferring to describe himself as “independent.” Although he did not intend it, his realistic style was a new stimuli and major driving force on the Impressionist movement for future artists. It would not be possible to describe the impressionistic approach to art and reality in its full complexity without referring to him. Claude Monet (1840–1926), Camille Pissarro (1830–1903), and Auguste Renoir (1841–1919) are typical followers of Manet. They were crucial in attracting many artists, such that their style ultimately proved highly influential in the development of Impressionism [44]. In this context, Fig. 4(b) shows that the core artists of French Impressionism were close to each other in terms of their social status, influence, and attitude toward artistic world.

In Fig. 4(c), the flattening part around Monet is changed into two “M+” extrema—Alfred Sisley (1839–1899) and Armand Guillaumin (1841–1927). Meanwhile, the arrow named Cezanne is an “m-” extremum, which plays an important role in dividing the single “M+.” Cezanne is said to have formed a bridge between Impressionism and the early 20th century’s new line of artistic enquiry, and to have created wonderfully idiosyncratic forms of Post-Impressionism. Given his prominence in these groundbreaking genres, Fig. 4(c) clearly shows the innovative role of Cezanne in French Impressionism.

Fig. 4(d) exhibits the inflow of this new style by adding a protrusion to the network. Vincent van Gogh (1853–1890) and Paul Gauguin (1848–1903) are the “m-” extrema, whereas Georges-Pierre Seurat (1859–1891), and Paul Signac (1863–1935) are the “M+” extrema. During the 1880s, some of the original Impressionists consolidated their advances, while the younger generation forged ahead with new developments and discoveries. Gauguin and van Gogh were forging ahead with Post-Impressionism, while Seurat and Signac embarked on a path that would lead them to develop a new style called Neo-Impressionism. Considering these facts, Fig. 4(d) effectively shows two mainstreams of French Impressionism. In addition, we determine the stylistic difference between Post- and Neo-Impressionism as being the fact that Neo-Impressionism used more short and broken brushstrokes called Pointillism.

The final step indicates the development of French Impressionism after the 1890s. In Fig. 4(e), Henri de Toulouse-Lautrec (1864–1901) and Charles Angrand (1854–1926) are the “m-” extrema, whereas Maximilien Luce (1858–1941) and Henri-Edmond Cross (1856–1910) are the “M+” extrema. By the early 1890s, Impressionism was a firmly established feature of the art landscape and was continually attracting new followers. Lautrec and Angrand played roles as forerunners of this new movement, and they were some of the most impressive artists of Post-Impressionism. Lautrec painted and exhibited together with van Gogh, influencing each other’s work [45], [46]. Angrand was also influenced by van Gogh’s thick brushstrokes and Japanese-inspired compositional asymmetry. Meanwhile, Luce and Cross joined the artistic world of Neo-Impressionism, becoming friends with luminaries such as Seurat and Signac. The two artists adopted their technique of Pointillism influenced by Signac and produced some of the most significant examples of Neo-Impressionist paintings.

Consequently, Fig. 4 provides an explanation for each of the artists roles in diffusing French Impressionism and how French Impressionism maintained the diversity of artistic style. French Impressionism developed by the repetitive appearance of leading artists such as Cezanne, van Gogh, and Seurat, which attracted a number of other artists. This visualization reflects the evolution through repeated bifurcation that demonstrates the growing autonomy and the virtue of endogenous innovation by the artists.

## IV. General Modeling of Network Evolution

To provide a rigorous mathematical evaluation, this study incorporates the concept of radial frequency (RF) for simulating the evolution of a network. RF patterns have been successfully used to investigate aspects of shape processing [47]. The relatively simple mathematical definition of RF patterns and the ease with which they can be generated and modulated has made

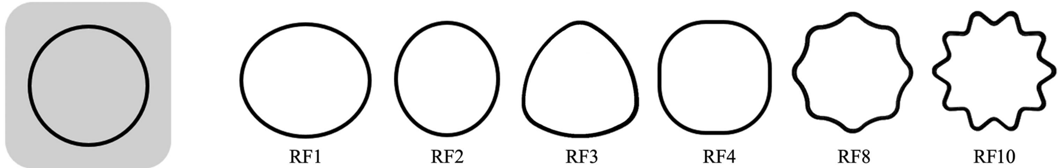


Fig. 5. Examples of the radial frequencies.

RF patterns a popular method in imaging studies [48]. RF is a circular contour with a cross-sectional luminance profile defined by a radial fourth-order derivative of a Gaussian (D4).

$$\mathrm{D4}(\gamma) = C\left(1 - 4\left(\frac{\gamma - \gamma_0}{\sigma}\right)^2 + \frac{4}{3}\left(\frac{\gamma - \gamma_0}{\sigma}\right)^4\right) \times exp\left(\left(-\frac{\gamma - \gamma_0}{\sigma}\right)^2\right) \quad (6)$$

In Eq. (6), $\gamma$, $C$, $\gamma_0$ , and $\sigma$ denote the radius, pattern contrast, mean radius, and peak spatial frequency, respectively. The base circles are deformed by applying a radial sinusoidal modulation to the radius $\gamma_0$ in Eq. (6) such that the radius of the deformed pattern at polar angle $\theta$ in radians is expressed by

$$\gamma(\theta) = \gamma_0(1 + Asin(\omega\theta + \emptyset)) \quad (7)$$

In Eq. (7), $\gamma_0$ , $A$, $\omega$, and $\emptyset$ denote the mean radius, amplitude of the sinusoidal modulation, radial frequency, and phase of the modulation, respectively. As shown in Fig. 5, RF patterns are highly controllable stimuli with which to represent and study the processing of simple shapes such as triangles (RF 3), squares (RF 4), and octagons (RF 8) [49], [50]. For example, in the Dutch artists network evolution (Fig. 3), the shape of the networks changes from RF 3, to RF 4, to RF 6. Because the final shape (e) has not changed compared to the previous network (d), RF 6 is maintained. In contrast, in the French Impressionists network evolution (Fig. 4), the shape of the networks changes from RF 4, to RF 5, to RF 7, to RF 8. Consequently, we can determine that the French Impressionists network is more dynamical than the Dutch artists network. Through the variation of the RF component, we can easily estimate the evolution of a concentric-circle network and compare the subtle differences between network dynamics using statistical values.

Furthermore, RF is useful to suggest more general evolution laws instead of being limited to specific cases. Generally, the data-driven network structure has no regularity because node attributes always change according to the applied data collection and analysis method. Hence, the evolutionary process is modeled off each network rather than formulating a theory of the network topology. However, RF can contribute to the understanding of the underlying topological structure of networks related to information diffusion. For any type of network evolution, new information plays an important role in maintaining the network as an information repository because it always creates new nodes, edges, and clusters. However, too frequent an inflow of information is not always better. It takes a reasonable amount of time for certain information to be fully shared with the people in a network. Time plays an important role in creating a robust cluster of people who think the information is meaningful. If new information continues to inflow, even before each cluster is formed properly, each protrusion will not be able to grow sufficiently. Furthermore, too many shallow protrusions finally returns to the circle. Simply put, the balance between bifurcation and continuation is important for maintaining network evolution. In this respect, we can view the historical fact that the Dutch artists network lasted longer than the French Impressionists network in a new light. Ultimately, this modeling using RF and evolution laws can be effectively used to predict the sustainability of network quantitatively.

## V. Discussions and Future works

In this study, a new visualization tool is proposed to build intuition about network dynamics and evolution effectively. Rather than treating the networks as insubstantial structures, this study views them as both dynamical and definable structures according to node attributes. As a results, this study highlights a pattern of network evolution that may have remained unnoticed through other traditional methods. This approach could help us to understand the embedded knowledge of dynamics in the process of network evolution owing to the following contributions:

First, the simplest visualization of the essence of the network evolution is presented. By fixing nodes and relations in artificial positions, the creation of a topological network space reveals more information about both the overview and the essence of the network. In particular, this study suggests how each node constitutes the evolutionary process of retention and variation in network topology. Using both Dutch artists and French Impressionists networks, this study offers examples of how data visualization will be used in evaluations to aid understanding, identify interesting pattern, and explore information from multiple perspectives. Furthermore, the reliability of the method is proven as the two networks show the process of evolution in agreement with the current knowledge of artistic developments.

Second, we provide insight into the sustainability of a network, which indicates that adequate diversity of information plays an important role in maintaining the network. For example, Dutch art is remarkable for its long duration compared to other art movements, whereas Impressionism lasted relatively short periods. This difference has been interpreted in terms of historical events or circumstances at that time. No one has interpreted that the difference may be caused by a network that controls the inflow and communication of information. However, comparing the Dutch artists network to the French Impressionists network, we can see that the Dutch network evolved by intersecting between bifurcation and continuation, whereas the French Impressionists network evolved by repeated bifurcation. The crucial difference between them is the frequency of information inflow that affects not only network topology but also node behavior. This is a common phenomenon in real-world networks, and it is not limited to the arts network. In summary, we showed that proper divergence of information ("bifurcation") is important to expand the network, whereas proper convergence of information ("continuation") is essential for the stability of the network.

Third, a major challenge for this study was appropriately defining the stationary shape of a network and dividing it into the stages of evolution. This was challenging because previous studies of evolutionary networks were constrained by a lack of powerful quantitative analysis. One reason for this problem is that network analysis only provides answers under certain conditions that do not involve a measure of change [51]. Although the records of large-scale networks are very rich, there has been no systematic attempt to classify the process of an evolutionary network itself. Under this circumstance, this study is a good example of practices that can be used to articulate practices for network evolution. Based on the quantification of node attributes, we show that a good visualization can provide various clues as to where a new node is likely to appear and where a new path is likely to emerge. In particular, the five-step visualization presents a considerable amount of information in an extremely economical and accessible way, thereby offering a new approach for the estimation, evaluation, and prediction of network evolution.

Fourth, the visualization results were applied to modeling, whereas general studies utilize the results of modeling for network visualization. The quality of visualization is often evaluated by how well the structure is preserved in the data. However, when the structure is unknown, it is necessary to use some quantitative measures of the visualization quality. This study thus formulated a five-step process for defining network topology with optimization of quantitative criterion and proposed a basis for the general modeling of network evolution. Using an easier version of the circular layout, we increases the readability of visualization. This optimization helped to identify a major change in network topology by displaying a recognizable transform of the network shape. Furthermore, we introduced RF components to provide useful information for network modeling, even though the RF algorithm has never been introduced into network analysis. By incorporating the results of visualization and the RF component, a more holistic and intuitive modeling is assigned to the evolutionary network. Consequently, the three-pronged approach—network analysis, quantitative method, and topological modeling—is expected to provide a revelatory insight into the principle of network evolution.

There are several directions for future work. First, various node attributes will be considered. The present method is designed to be applicable when there are quantifiable node attributes such as color, brushstroke, and texture, which are common elements in all paintings and characteristic features for distinguishing each artist. The crucial prerequisite in applying the method is to include numerical data for defining the distinct characteristics of a node. For example, musician networks could be analyzed because music has attributes such as tempo, pitch, articulation, dynamics of every note, and rhythms that are often handled as one-dimensional sequential data. Both painting and music have the same task that challenge the modeling of the complex and expressive performance with computational methods. Therefore, each musical composition can be measured quantitatively and arranged in a historical frame or stylistic evolution. To determine the applicability of this method, we will continue to evaluate new attributes of nodes by conducting more case studies, and we will apply this method to other application fields. Although there is much work to do, this style-centric visualization method and its applications will help organize our thinking of network evolution.

The second consideration is the ordering of the nodes. In many cases, being able to explain the prevalence of network topology depends on the order in which the node is introduced. In the artists network, the period of intense activity was used as the obvious criterion of node ordering, but different criteria may be required for ordering depending on the properties of the network. Through various case analyses, we will find and match the ordering method that fits the various characteristics of node attributes. In addition, if we could measure the intensity of each extremum such as protrusion and indentation, more sophisticated visualization and modeling of the network would be possible.

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